Announcements

1) EC problem on Mean Value theorem due date extended to next week. (Tuesday)

2) New EC problem: bonus from Exam 2 (get 1/2 the points you missed) (Thursday)

3) EC survey on Lutzer's "Calculus" Chapter (2 points)

L'Hopitals Rule

or "Making Limits Easy"

Note. This is not in Stewarts Single Variable Calculus, volume 1.

The Idea:

When is $\lim_{x \to a} \frac{f(x)}{g(x)}$

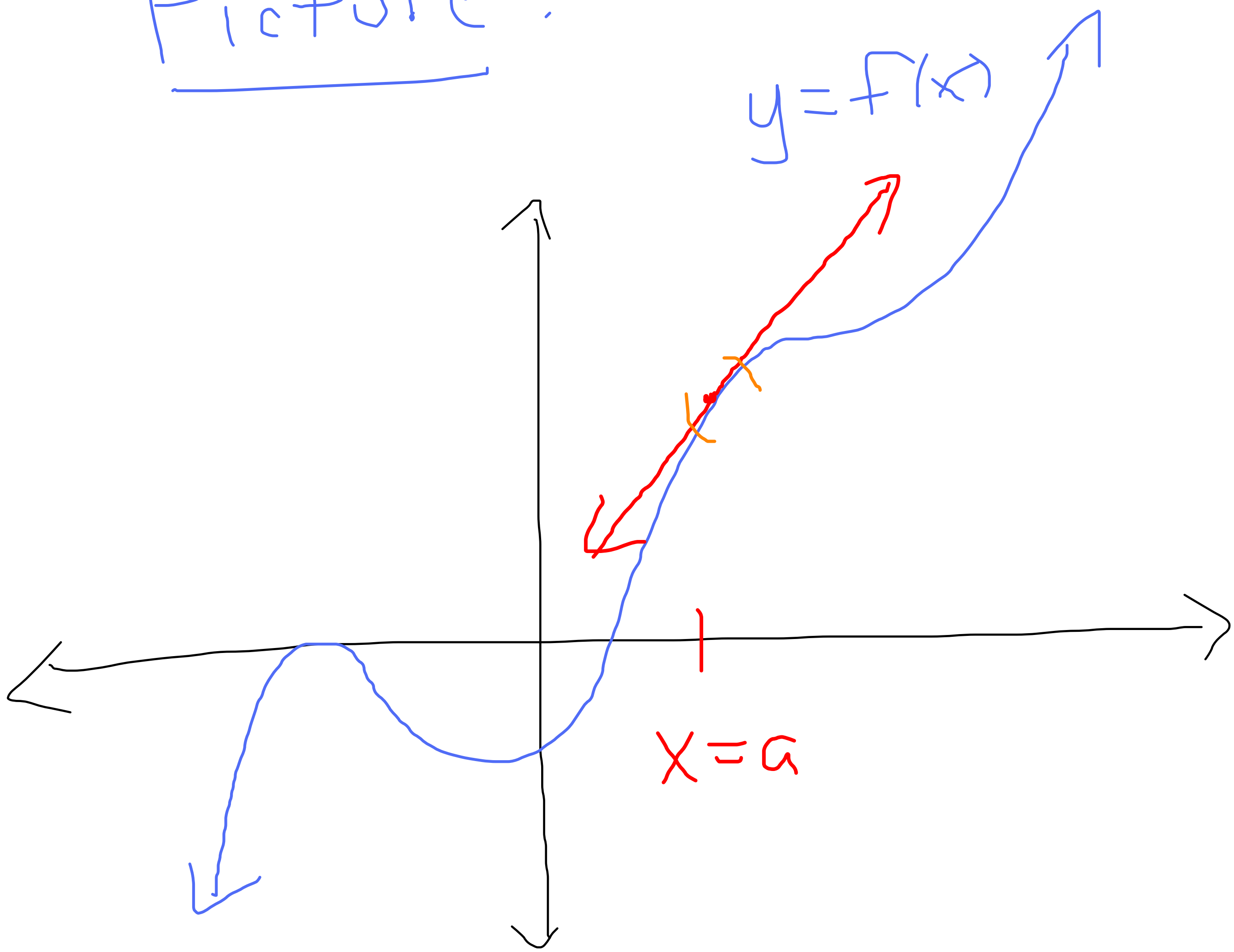
a pain? $\frac{0}{0}$

Also, $\frac{\pm\infty}{\pm\infty}$.

How to address these limits?

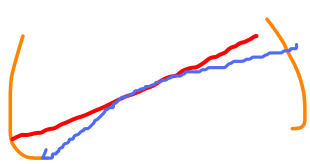
As $x \to a$, if $f'(a)$ exists, then the graph of f "is approximated" by the graph of the tangent line at $x=a$

Picture:



red line = tangent line

zoom in on orange interval



In this small region,

f is "like" the tangent

line $y - f(a) = f'(a)(x-a)$.

Similarly, if $g'(a)$ exists,

g is "like" $y - g(a) = g'(a)(x-a)$

so $\dfrac{f(x)}{g(x)}$ is "like"

$$\frac{f'(a)(x-a)+f(a)}{g'(a)(x-a)+g(a)}$$

Dividing out by $(x-a)$

gives

$$\frac{f'(a) + \frac{f(a)}{x-a}}{g'(a) + \frac{g(a)}{x-a}}$$

"like" $\dfrac{f'(a)}{g'(a)}$

(if $f(a) = g(a) = 0$)

L'Hopital's Rule

Suppose $\left(\lim_{x \to a} f(x) = \lim_{x \to a} g(x) = 0 \right)$

or $\left(\lim_{x \to a} g(x) = \pm\infty \text{ and } \lim_{x \to a} f(x) = \pm\infty \right)$.

Then if f' and g' exist in an open interval containing $x = a$ (except possibly at $x = a$),

$$\boxed{\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}}$$

Example 1: $\lim_{\theta \to 0} \frac{\sin(7\theta^2)}{11\theta^2}$

$\lim_{\theta \to 0} \sin(7\theta^2) = 0$

$\lim_{\theta \to 0} 11\theta^2 = 0$

So use l'Hopital's Rule, indicate you are using it.

$$\lim_{\theta \to 0} \frac{\sin(7\theta^2)}{11\theta^2}$$

$$\overset{L'H}{=} \lim_{\theta \to 0} \frac{\cos(7\theta^2) \cdot 14\cancel{\theta}}{22\cancel{\theta}}$$

$$= \lim_{\theta \to 0} \cos(7\theta^2) \cdot \frac{14}{22}$$

$$= 1 \cdot \frac{14}{22} = \boxed{\frac{7}{11}}$$